ORAL LANGUAGE AND THE APPROXIMATE NUMBER SYSTEM – A PRELIMINARY STUDY

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Abstract

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The approximate number system (ANS) involves the processing of rudimentary quantity and is thought to be an innate developmental building block for mathematics and its sister construct, the symbolic system. The conventional belief is that the ANS is language independent; however, this notion is questioned and explored in the current study, which represents a preliminary investigation into the concurrent and longitudinal relations between different aspects of oral language and the ANS in 4-year-old children and one year later when they were 5. A sample of 26 children (13 boys;13 girls) with average intelligence completed standardized measures of oral language and verbal memory, and a computerized quantity discrimination task that required children to accurately discern between two visually presented quantities. Correlational analysis showed concurrent and longitudinal relations between different aspects of language and quantity discrimination. This suggests that different aspects of language predict quantity discrimination over a one-year period and challenge the current and accepted theory that the ANS is a language independent system. The findings also have implications for early childhood education – avenues to strengthen a child's ANS via targeted oral language instruction, curricula, and subsequent provision of experiences. The findings also support early oral language screening to monitor or provide opportunities for improving quantity approximation skills. This early intervention could impact later symbolic processing and mathematic success.

Keywords: Non-symbolic system, approximate number system, quantity discrimination, symbolic system, oral language, morphology, syntax, semantics, relational concepts.

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Overview

Quantitative knowledge consists of two foundational systems: 1) the symbolic system, which involves the Arabic number system (Purpura & Napoli, 2015), and 2) the non-symbolic system, which develops without formal teaching, precedes symbolic skills, and is commonly attributed to the Approximate Number System (ANS; Dehaene et al., 1998; Furman & Rubinsten, 2012). The ANS is believed to be language independent, innate (Feignenson et al., 2004; Lindskog et al., 2013), and the foundational system of processing that underlies the ability to interpret, represent, and compare quantities (Halberda & Feigenson, 2008). The ANS is also thought to play a crucial role in enhancing mathematics learning performance in the symbolic system (Hyde et al., 2014) and to be a predictor of mathematics achievement in school (Hyde et al., 2014; Libertus et al., 2011). The current literature has established that improving language skills can also improve mathematics achievement (Grimm, 2008; Toll &Van Luit, 2014) and is a strong predictor of early mathematical success (LeFevre et al., 2010; Purpura et al., 2011). The conventional belief is that the ANS is a language independent system. However, since we know that the ANS develops over time, is a predictor of later mathematics achievement, and that early language skills have been shown to affect its sister construct – the symbolic mathematical system, we can reasonably postulate by extension that oral language may in fact influence the ANS. The goal of the current study is to conduct a preliminary examination of the longitudinal relation between oral language and the ANS in young children.

Quantitative Knowledge

Quantitative knowledge is what we commonly associate with the mental processing of quantity. Quantitative knowledge is conceptualized as being constructed of the cognitive processes and abilities encompassed by two core systems – the symbolic system and the non-symbolic system (Dehaene et al., 1998; Furman & Rubinsten, 2012; Purpura & Napoli, 2015). The symbolic system assigns symbols to specific amounts (e.g., Arabic numeral 10 to represent the quantity of 10 objects), math functions (e.g., "+" or "-" to indicate addition or subtraction, respectively), and mathematical sequences (e.g., the multiplication pattern 3, 6, 9, 12). The non-symbolic system does not involve symbols for precise quantity discrimination, but is thought to depend on an innate ability to approximate amounts (Dehaene, 1992; Geary et al., 2015).

Symbolic System

The symbolic system includes mathematical abilities such as: verbal counting, enumeration (the use of a count list in a counting context), digit recognition, the understanding of cardinality (the total number of items in a set), and learning to associate number words with their corresponding Arabic numerals (Baroody, 2003; Chu et al., 2015; Clements, 2007; Clements & Sarama, 2004; Geary & vanMarle, 2016; Van Herwegen et al., 2018). Symbolic numerical magnitude is another element of the symbolic system that is the ability to understand, estimate, and compare sizes or fractions using Arabic numerals or number line estimation (Fazio et al., 2014). For example, the ability to determine which of two Arabic numerals is larger: 5 or 8; 1/2 or 1/3, and estimating the number position of 500 (midpoint) when given a horizontal number line ranging from 0 to 1000.

The symbolic system is comprised of three primary and overlapping language dependent phases: informal numeracy, numeral knowledge, and formal numeracy (Purpura & Napoli, 2015; Purpura et al., 2013). Informal numeracy consists of skills learned prior to schooling. This begins with comparing exact sets to distinguish quantities (e.g., two oranges are the same quantity as two apples), moves to the verbal number word sequence (e.g., one, two, three), and is followed by connecting the verbal counting sequence to fixed sets (e.g., two apples and two oranges), connecting number words with quantities (e.g., the word “two” represents two items), and subitizing (i.e., the ability to visually recognize the number of items in a small set such as three without counting). Informal numeracy also includes the ability to manipulate quantities to form new quantities with learned knowledge (e.g., basic addition and subtraction through word problems; Baroody et al., 2013; Purpura & Napoli, 2015; Purpura & Lonigan, 2013). The numeral knowledge phase involves the mapping of skills and abilities learned in the informal numeracy phase onto the Arabic numeral system while connecting these numbers to their respective quantities. The third phase, formal numeracy, consists of mathematical concepts and skills learned through formal instruction, typically in school, and over time such as operations employed to solve problems using number equations (e.g., addition and subtraction problems such as 2 + 3 =), place-value, and knowledge of the base-ten and decimal systems (Purpura & Napoli, 2015).

Non-Symbolic System

The non-symbolic system involves the cognitive processing of rudimentary quantity that predates the emergence of language (Agrillo et al., 2013; Dehaene, 1992; Halberda et al., 2008; Hyde et al., 2014). It is believed that these cognitive processes may also exist in non-humans and across species to some degree, serving an evolutionary function suggested to be imperative for survival (Geary et al., 2015). For example, an animal’s ability to differentiate a large cache of food from a small one would directly affect its foraging and energy efficiency and therefore its chances of survival. Proposed to be present from the very early years in life, the non-symbolic system eventually gives rise to the skills and abilities characteristic of the symbolic system’s informal numeracy stage and the subsequent development of numeracy (Dehaene, 1992;

Halberda et al., 2008; Starr et al., 2013). This non-symbolic preverbal and “intuitive” number sense is suggested to be a developmental building block for mathematics that may facilitate the acquisition of numerical symbols and later mathematic abilities (Hyde et al., 2014; Starkey & Cooper, 1980; Starr et al., 2013). There is general consensus that numeracy develops from early approximate quantity discrimination skills that are purported to be present in infancy – this is the ability to perceive, understand, and manipulate quantity (Dehaene, 1992; Leibovich et al., 2016). For example, infants as young as 6 months old are reported to be able to discriminate visual arrays and auditory sequences between large sets of elements on the basis of numerosity (e.g., 8 vs. 16 dots or 8 vs. 16 sounds; Wood & Spelk, 2005).

It is suggested that non-symbolic number representations are core knowledge, which operates on two common underlying cognitive mechanisms in infants and adults (Agrillo et al., 2013; Wood & Spelk, 2005). The object tracking system is the first of these mechanisms, which is believed to represent and allow the tracking of individual elements thought to be utilized to enumerate precise small quantities (usually 3 to 4 items) that supports subitizing (Agrillo et. al., 2013.) The other underlying mechanism for non-symbolic number representations is the operation characteristic of the ANS (Agrillo et. al., 2013) – the focus of the current study.

Approximate Number System (ANS)

The ANS is the intuitive and abstract cognitive processing that allows us to estimate and represent number when we are presented with visual or auditory stimuli (Feigenson et al., 2004; Odic & Starr, 2018; Wood & Spelke, 2005). When quantity information is presented visually, we instinctively extract and represent the approximate number of items in the scene – like scanning a gymnasium full of people for example, and being able to approximate the number of people or the ratio of males versus females in the room (Odic & Starr, 2018). The estimate we extract is imprecise, yet it is highly efficient and used for multiple number tasks and for daily decision making, such as deciding how many bags might be needed to hold the groceries in our cart or what grocery line up would facilitate a quicker service (Bonny & Lourenco, 2013; Chen & Li, 2014; Odic et al., 2016; Odic & Starr, 2018; Soto-Calvo, 2013). Approximate numerical skills are generally believed to depend on an internal mental number line or continuum that is automatically initiated when numerical information is presented (Dehaene & Cohen, 1991; 1995; Soto-Calvo, 2013). When the ANS is activated by the presence of numerical information, small numerical amounts are represented on the mental continuum and imprecision increases proportionally as the amount increases (Dehaene & Cohen, 1991, 1995; Hyde et al., 2014; Soto-Calvo, 2014). ANS processing is also believed to be utilized to perform non-symbolic arithmetic operations (e.g., addition and subtraction) and supports other diverse computations such as numerical comparison and ordinal comparison (Gilmore et al.,2010; Hyde et al., 2014).

ANS Performance

Two hallmark performance characteristics have consistently been observed when the ANS is in use: 1) As the number of objects presented in a visual scene increases, the numerical estimates we make become more variable and 2) When discriminating between two sets to discern which set is greater in quantity, the accuracy of this estimate is ratio dependent (Halberda & Feigenson, 2008; Odic & Starr, 2018). For example, it is harder to distinguish between 70 and 80 items (7:8 ratio) than it would be for 70 and 35 items (2:1 ratio). Although the degree of preverbal infants' numerical abilities and abstractness of their numerical representations are debated, studies (using habituation/dishabituation methodology) show that infants appear to discriminate approximate quantities between sets of items, but do so with less precision compared to older children and adults (Brannon, 2002; Dehaene et al., 2004; Halberda &

Feigenson, 2008; Xu & Spelke, 2000; Xu et al., 2005). For example, a 6 month-old infant who is presented with two streams of images, one showing 8 dots continuously and the other alternating between 8 and 16 dots, demonstrates quantity discrimination if they look longer at the alternating stream (interpreted as recognizing that a difference exists between the quantity of dots in the alternating images) controlling for non-numerical stimulus dimensions like cumulative surface area, dot size, and array density (Wang et al., 2018). Research shows that infants can discriminate 1:2 ratio arrays (e.g., 4 vs. 8; 8 vs. 16), but are not able to discriminate 2:3 ratios (e.g., 8 vs. 12; 16 vs. 24; Halberda & Feigenson, 2008; Libertus & Brannon, 2009). However, by 9 months of age, ANS acuity (the ability to discern a difference between two quantities beyond an arbitrary level of accuracy) improves and infants have been found to differentiate 2:3 ratios in both visual and auditory domains (Halberda & Feigenson, 2008). In adulthood, the smallest numerical ratio consistently identifiable has been 7:8 (Halberda & Feigenson, 2008). There is general consensus that the ANS is refined or improved over the life span and full acuity does not occur until quite late in development (reaching adult levels during preteen years, but gradually increasing up until age 30), and after formal mathematics instruction has taken place (Chen & Li, 2014; Halberda & Feigenson, 2008; Libertus et al., 2016).

Measurement of ANS Performance

ANS performance across the lifespan is typically tested in individuals by briefly flashing sets of dots (or sequential tones) so they cannot be counted. Individuals are asked to compare one set to a different set of dots, Arabic numbers, or a number word (e.g., *five*) presented simultaneously or concurrently (Dietrich et al, 2015; Odic & Starr, 2018). As the ratio between the quantity of two sets increases (e.g., from 3:4 to 1:2), the difference between the two sets is more apparent; this is known as Weber’s law. Weber’s law is useful for describing ANS performance at the point at which the level of accuracy between ratios begins to change (i.e., threshold), and an individual's ability to distinguish between two sets of quantities is identifiable (Bonny & Lourenco, 2013; Dehaene, 1997; Chen & Li, 2014; Odic & Starr, 2018). Reaction time and error rates on measures of the ANS decrease as the ratio of quantities increase (Bugden, DeWind, & Brannon, 2016). For example, a 1:6 ratio is more easily differentiated than a 1:3 ratio and a 1:3 ratio is even more easily differentiated than 3:4 or 2:3 ratios (Wang et al., 2018).

It is useful then to be able to objectively measure an individual's ability to consistently identify a difference between two quantities. The smallest consistently identifiable difference between two quantities represents an individual's ANS acuity (Bonney & Lourenco, 2013; Chen & Li, 2014; Odic & Starr, 2018; Wang et al., 2018). A numerical measure of ANS acuity can be calculated by taking the difference between the two quantities being compared (e.g., 1:2 or 7:8) and dividing by the smaller number (e.g., 2 – 1 / 1 = 1.0 or 8 – 7 / 7 = .14). This is referred to as a Weber fraction (*w*; Chen & Li, 2014; Park & Starns, 2015; Grantham & Yost, 1982). A smaller Weber fraction (e.g., .14 vs. 1.0) means that finer differences between quantities can be consistently identified and that the individual has greater ANS acuity (Mazzocco et al., 2011; Park & Starns, 2015). Developmentally, the Weber fraction appears to decrease with age from 1.0 in infancy to .14 in adulthood (Halberda & Feigenson, 2008). However, in studies where participants, typically young children, show widely varying performance even within one ratio, the Weber fraction cannot be applied. This is because young children do not often demonstrate a consistent minimum threshold of detection – or being consistently correct on all the trials within one ratio. Therefore, for young children, calculating accuracy (percent correct) to represent ANS acuity provides a better description of a child's ability to differentiate between two quantities than Weber's fraction (Geary & vanMarle, 2016; Honore & Noel, 2016; Inglis & Gilmore, 2014;

Wang et al., 2016).

For example, to measure the minimum threshold of quantity identification for one individual in a series of increasingly small ratio differences (i.e., 1:4, 1:3, 1:2) where there are four items for each ratio on a test, a participant would have to perform accurately on 3 of the 4 items representing a single ratio. The participant would also have to score at least 75% correct on all the preceding larger ratios in order to be able to assign the smallest ratio at 75% correct as the minimum threshold for detection. Using the above test example, if a participant scored 50% correct for the 1:2 ratio, 75% correct for the 1:3 ratio, and 75% for the 1:4 ratio, the 1:3 ratio would be considered the minimum threshold of quantity identification. However, if this participant scored 50% correct for the 1:4 ratio, a consistent minimum threshold for detection cannot be established, and therefore a Weber fraction could not be calculated.

Some researchers found the Weber fraction to be less reliable than an overall accuracy measure of performance (Bonny & Lourenco, 2013; Geary & vanMarle, 2016; Inglis & Gilmore, 2014). It is suggested that young children may experience fatigue, lack of attention, may lose motivation, or guess randomly while test taking which may make it less likely that consistent minimum threshold results are obtained (Honore & Noel, 2016). It is also possible that there is no finite minimum threshold of quantity identification for the ANS. In these studies, accuracy (percent correct) was calculated using a computerized quantity discrimination task with ratios ranging in difficulty that required the selection of the larger quantity over multiple trials. Being able to score and measure ANS performance reliably in children facilitates the use of this measurement to track ANS development and look for significant associations with mathematical performance. Research with the ANS generally focuses on whether ANS performance in children relates to their early mathematics development, symbolic mathematics performance, and later mathematics achievement (beyond early childhood). This literature shows strong evidence supporting a correlation between the ANS and mathematics achievement (Bonny & Lourenco, 2013; Geary & vanMarle, 2016; Honore & Noel, 2016; (Odic et al., 2015; Wang et al., 2016).

ANS and Mathematics Achievement

There is evidence and several theories proposed in the literature regarding the importance of the ANS as the foundation for more sophisticated symbolic mathematics and later success in mathematics achievement. ANS acuity in early childhood has been found to be correlated with later performance in mathematics – both in cross sectional data and longitudinally (Bonny & Lourenco, 2013; Chen & Li, 2014; Fazio et al., 2014; Halberda & Feigenson, 2008; Libertus et al., 2011; Mazzocco et al., 2012; Wang et al., 2016). For example, a study investigating ANS acuity and achievement in formal mathematical tasks showed that accuracy of quantity discrimination in kindergarten was related to arithmetic achievement in first grade and math fact retrieval two years later (Desoete et.al., 2010). Libertus et al. (2011) also investigated the link between the ANS and math ability prior to formal mathematics instruction. They gave very young children (3 to 5 years old) a math task that did not require symbolic use or arithmetic calculations and found a positive association between the ANS and math ability, even when age and verbal skills were controlled for. Research with a gifted adolescent sample suggests that the relation between ANS acuity and mathematics performance exists even at a high level of mathematics achievement (Wang et al., 2017). Corroborating evidence supporting the importance of ANS acuity to learning in mathematics comes from studies that show ANS acuity differentiates children with dyscalculia (mathematical learning disability) from their peers with typical mathematics skills (Mazzocco et al., 2011; Price et al., 2007).

It has been proposed that the sharpening of ANS acuity may be due to both maturation and experience (i.e., practice at quantity discrimination; Halberda & Feigenson, 2008). In support of this finding, research with children with math learning disabilities who were given eight hours (over five weeks) of quantity discrimination practice (designed to engage the ANS) showed improvement on both non-symbolic and symbolic numerical tasks (Halberda & Feigenson, 2008). Similarly, an investigation into whether approximate arithmetic training (approximating or comparing quantities) in the form of a computerized game positively impacted informal versus formal mathematics ability in preschool children showed rather unexpectedly that ANS acuity training significantly improved informal math performance in children with low math scores (Szkudlarek & Brannon, 2018). This finding supported the premise that approximate arithmetic training is especially effective for children with lower mathematics skills over those with a higher level of skill (Szkudlarek & Brannon, 2018).

Recent research exploring attempts at improving ANS acuity through educational training and numeracy instruction have reported positive results, which highlights the essential link between the ANS and later symbolic mathematics achievement, its importance to overall learning, and the potential to detect math learning problems and potentiate intervention opportunities (Chen & Li, 2014; Piazza et al., 2013). For example, studies training children in approximate arithmetic (dot collection comparisons) demonstrate that the training improved symbolic exact arithmetic performance (Honore & Noel, 2016; Odic & Starr, 2018). Further, Park and Brannon (2013) demonstrated that training the ANS with addition and subtraction of arrays of dots improved symbolic addition and subtraction, supporting the hypothesis that complex math skills are linked to the ANS. Similar results were found by Van Herwegen et al., (2017), which adds credence to the body of research suggesting that training the ANS can lead to improvements in symbolic math skills (Van Herwegen et. al., 2017). As a result, how the ANS and the symbolic system support one another and the underlying mechanisms that facilitate the relation between the two in learning and performing mathematics has been the focus of much research in recent decades.

ANS and the Symbolic System

The ANS functions support the ability to enumerate quantities in an imprecise way and helps with approximate addition and subtraction, whereas symbolic system skills help us to represent exact symbolic numbers (e.g., “20” or “twenty”; Bugden et al., 2016; Gilmore et al., 2010; Hyde et al., 2014). How the initial preverbal representations of quantity form the basis from which children’s first number words (informal symbolic knowledge) develop is debated, however, it is generally agreed that before the age of 4 (prior to adopting Cardinality) children map symbols and words onto their ANS and preexisting sense of number and quantity (Bonny & Lourenco, 2014; Bugden et. al., 2016; Odic et al., 2015). At this point in development, the relation between the ANS and the symbolic system may be described as directional where the ANS (approximation, quantity discrimination, understanding of magnitude) exists first and then the symbolic system superimposes symbols with learning and experience (Odic et. al., 2015). After 4 years of age (after adopting Cardinality), children gradually gain greater knowledge of numeric symbols, words, and arithmetic skills and operations and the relation between the two systems becomes bidirectional and the symbolic system enhances ANS acuity (Bugden et. al., 2016). In this view, we learn formal mathematics from the layering of previously acquired symbolic number notations and knowledge of quantitative concepts and operations which were initially built upon non-symbolic representations of numerosity in the ANS (Bonny & Lourenco 2014; Dehaene, 1992; Odic & Starr, 2018; Wong et al., 2016). For example, a child with the ability to identify quantities more precisely via the ANS may find it easier to learn and comprehend number words, which in turn, may facilitate early arithmetic learning (Odic & Starr, 2018). The ANS may continue to influence math cognition later in life with respect to relating magnitudes (e.g., 1, 2, 3, 4) to their ordinal representations (e.g., first, second, third, fourth; Odic & Starr, 2018).

When the ANS is utilized during a symbolic number task (e.g., addition) it is hypothesized that for some functions like estimating, remembering, and comparing, coordination between the ANS and other cognitive systems (e.g., language; Odic & Starr, 2018) is required. For example, visual or auditory stimuli (e.g., a tone) are represented and can be estimated either concurrently or from quantities that are stored in memory. These representations and estimations are with language using number words (e.g., five), held in short-term and long-term memory, divided into subgroups (e.g., colour, size), and used arithmetically (e.g., added and compared, smaller vs. larger; Odic & Starr, 2018). Understanding where ANS functions might occur anatomically within the brain relative to other cognitive functions can help us appreciate the interconnectedness between the ANS and other cognitive functions. Moreover, realizing what regions of the brain become activated simultaneously or exclusively when performing a mathematical task leads us to ask if the ANS is truly separate from the influence of language, as the current body of research suggests.

ANS and the Brain

Research in neuroimaging suggests that the ANS activates the parietal cortex and the intraparietal sulcus regions of the brain (regions integral for processing numerical magnitudes and calculations) when processing different ratios during quantity discrimination (Bonny & Lourenco, 2013; Dehaene, 2009; Izard et al., 2008). Although the intraparietal sulcus brain region is the major region of the brain associated with mathematical processing, there is also evidence that it is linked to language processing. The left side may act as an attentional modulator of neural networks that specialize in processing order or language representations (e.g., phonological, orthographic, and semantic processing) and the right may be involved in grammatical processing (e.g., quantifiers, determiner – noun phrases; Carreiras et al., 20109; Majerus et al., 2006). While the ANS appears to activate similar brain regions across development, there are shifts in the amount of neural activity for the brain regions activated by ANS involvement over time (Bonny & Lourenco, 2013; Bugden et. al., 2016).

In infants, the right side of the intraparietal sulcus is involved in the processing of non-symbolic magnitudes, but the parietal and prefrontal cortices (associated with executive function, attention and memory) exhibit high neural activity as well (Bonny & Lourenco, 2013; Bugden et. al., 2016; Miller et al., 2002). One study investigated ANS neural activity in the infant brain by changing the ratios of objects presented to an infant (i.e., images of sets of objects were presented with occasional changes in number or type of object) and showed that at three months, there was greater activation of the parietal and prefrontal cortices for both smaller quantities (e.g., 2 versus 3) and larger quantities (4 versus 8; 4 versus 12; Izard et al., 2008). These findings support that a system exists in infancy for representing both small and large numbers that occurs in specific brain regions that are engaged when developing number sense and learning (Izard, et al., 2008). The prefrontal cortical involvement at this early stage of development also implies that executive function (composed of three core functions – inhibitory control, working memory, and cognitive flexibility; Diamond, 201) supports the learning and representation of quantity. The literature shows that executive function also has an association with language and that the relation is complementary and recursive (Diamond, 2014; Muller et al., 2009).

In childhood, there is a shift in activation from the parietal cortex to greater involvement of the intraparietal sulcus, with continued activation in the prefrontal cortex (Ansari & Dhital, 2006; Bonny & Lourenco, 2013). The parietal and intraparietal sulcus brain regions are believed to be the locations for the processing of spatial manipulations, arithmetic (e.g., quantity calculations), and integrating sensory information (e.g., visual, somatosensory, auditory; Blakemore & Firth, 2005). The involvement of the prefrontal cortex reflects the use of greater working memory and attentional resources required for learning (semantic associations, symbolic numerals, solving problems; Bugden, et al., 2016). Thus, ANS activity in the brain is dynamic throughout development and shifts into other regions of the brain that provide other functions (symbolic math processing, executive function, and language) that support ANS processing, directly or indirectly. One study suggests that as symbolic numbers are learned (during numeral knowledge and formal numeracy phases of development) they activate the intraparietal sulcus more preferably in the left hemisphere (Bugden et al., 2016). It has been proposed that this increased activity may be the result of less reliance on attention and working memory resources as children make the transition from more controlled and effortful processing to more automatic processing of quantity (Ansari & Dhital, 2006; Rivera et al., 2005). Thus with age, there may be maturation of prefrontal cortical functions (e.g., attention, working memory) that support mental arithmetic skills in children (Rivera, et al., 2005). We may further deduce that language, evidenced by its complementary and recursive relation with executive function (Diamond, 2014; Muller, et al., 2009), may be particularly influential during the early years of non-symbolic (and symbolic) math development.

These developmental shifts in brain region activation suggest that there is some specialization of the intraparietal sulcus when processing non-symbolic numerical information (Ansari & Dhital, 2006; Bonny & Lourenco, 2013). Holloway and Ansari (2010), investigated the brain regions that support symbolic and non-symbolic numerical representations in children and adults to assess age related differences and found that a larger network of visual and parietal regions showed activation in adults, with an increased activation to both symbolic quantities (e.g., using Arabic numbers) and non-symbolic quantities (e.g., arrays of squares) in the right inferior parietal lobe near the intraparietal sulcus compared to children (Holloway & Ansari, 2010). This research corroborates the evidence of developmental shifting in brain activity according to region during the processing of mathematics over time showing greater and more focused activation in the intraparietal sulcus region. Movement of neural activity from one region to another in the brain over time during mathematical processing highlights the dynamic nature of cognitive networking, and the shared space and activity in the intraparietal sulcus for both mathematical systems suggests that they may be influential to one another.

Physiological evidence of shared anatomy and the dynamics between cognitive processes supports the relation between non-symbolic (e.g., quantity processing) and symbolic system functions that are intimately intertwined with language (Bugden, et al., 2016). Studies using functional magnetic resonance imaging (fMRI) looking at the intraparietal sulcus show that the bilateral intraparietal sulcus is a key neural substrate of both quantity and symbolic processing, with some areas within the intraparietal sulcus more biased toward number comparison (Ansari & Dhital, 2006; Bugden, et al., 2016; Holloway & Ansari, 2010). The research suggests that non-symbolic and symbolic quantities activate overlapping parietal regions (e.g., right and left) within and around the intraparietal sulcus (Bugden, et al., 2016). The neurophysiology of the intraparietal sulcus and functioning of non-symbolic and symbolic processing provides indirect evidence for the current thinking in the literature which proposes that the advancement of the mathematical mind is made possible due to the intertwined functions of both the left and right (non-symbolic and symbolic functions) intraparietal regions that have developed to characterize non-symbolic quantity (Bugden, et al., 2016). This evidence also supports the proposal that language may also influence the non-symbolic system given the high degree of association between language and the development of the symbolic system.

The areas of the brain utilized during ANS processing and its interplay with the rest of the brain over the course of development are important factors when considering possible influences on ANS ability over time. Studies investigating lesions of the parietal region have identified several associated pathologies including but not limited to: aphasia (the inability to comprehend or formulate language), dyscalculia, dyslexia, and agrapheasthesia (the inability to feel numbers or letters drawn on a hand with eyes shut; Culham & Valyear, 2006; Simon et al., 2002; Molko et. al., 2003; Steinman et al., 2010). These pathologies that effect both language and math abilities while involving a common brain region (parietal lobe) illustrate the complex and interconnected nature of human language and math cognition. While the current body of literature suggests that the ANS and non-symbolic system operates independently of language, studies in neurophysiology and brain lesions provoke reason to challenge this idea. Specifically, what influence does oral language have on the ANS and could it potentially be exploited to enhance mathematical learning? Examining the enmeshed relation between language and the symbolic mathematical system and the unfolding of skills during development provides support to postulate a language and ANS connection.

Language and Symbolic Mathematics

The current base of evidence in the literature regarding the relation between language and symbolic mathematics is expansive and continues to mount. It is now generally accepted that most aspects of symbolic mathematics have significant language components and that language skills directly and uniquely link to early numeracy skills, and mathematics performance and achievement (McClung & Arya, 2018; LeFevre, 2010; Purpura et al., 2016, 2017; Purpura & Napoli, 2015; Zhang et al., 2017). Improving language skills improves mathematics achievement (Grimm, 2008; Pupura, et al., 2016; Toll &Van Luit, 2014) and deficits in language skills required for reading and writing often lead to delays in numeracy development (Jordan et al, 2002; Purpura & Napoli, 2015). To fully appreciate the relation between language and the symbolic mathematical system, it is important to understand: 1) the components of language and 2) the literature examining language and symbolic mathematics.

Components of Language

Language is a dynamic system of communication involving: phonology (sound system), morphology (internal structure of words and how they are formed), semantics (word meaning and properties), syntax (rules for combining sentences), and pragmatics (verbal and non-verbal social rules; Im-Bolter & Cohen, 2007). All components of language are acquired simultaneously over development and used interactively via expressive (production) and receptive (comprehension) channels (Genishi, 1988; Im-Bolter & Cohen, 2007).

Phonology includes the production and understanding of speech sounds (or phonemes) in accordance to the rules of a particular language (Hedge & Maul, 2006; Mann & Ditunno, 1990). Phonological abilities include phonological awareness (awareness of the sound structure of language) and phonological processing, which requires the retrieval and manipulation of individual phonemes of speech (e.g., /i/, /a/), our verbal short term memory for coding information that is heard into temporary storage, and the efficient retrieval of that phonological information in our long term memory (Kalaiah, 2015; Kuzmina et al., 2017). Morphology is our understanding of the internal structure of words (Aronoff & Fudeman, 2010; Hedge & Maul, 2006). A morpheme can be a word that cannot be broken down (free morpheme or root word) like *jump* or it can be a small unit of meaningful language that combine with other morphemes, such as those that indicate verb tense (e.g., jump, jumped, jumping), word endings (e.g., ed, -ing, -s, -es), and affixes (e.g., re-, -ation) which alters the meaning of a root word (Aronoff & Fudeman, 2011; Hedge & Maul, 2006; Im-Bolter & Cohen, 2007). Semantics encompasses the features, meanings, and properties of concepts (e.g., vocabulary, understanding concepts associated with and among words) that one learns gradually overtime (Hansson et al., 2015; Honig, 2007). This includes global knowledge of objects (e.g., stars, energy), events (e.g., super nova) and the relations among them (e.g., furniture, fruit). The aspect of language represented by rules that govern word combinations and sentence structures is called syntax (Hedge & Maul, 2006; Honig, 2007). Syntactic rules are unique to each language and determine what order words can appear in a sentence (Hedge & Maul, 2006; Honig, 2007). For example, in the English language, a complete sentence must contain a subject and a predicate (i.e., noun and verb; Hedge & Maul, 2006). Knowledge of syntactic rules develops rapidly and by the age of 6 or 7, children make very few errors in the production of acceptable basic sentences (Honig, 2007).

The body of literature on the relation between language and mathematics focuses mainly on a few components of language, such as phonology, semantics, and syntax or discusses language as a general construct. The research on mathematics tends to focus on specific skills (e.g., subsidization, enumeration) in a particular strand of mathematics (e.g., geometry, numeracy) over different stages in development or at a specific age point, or looks at mathematics as a general construct. Understanding the components of language in relation to mathematics facilitates our ability to appreciate how language may integrate, directly or indirectly, with the mathematical domain.

Language and Symbolic Mathematics

Phonological awareness, semantics, and syntax, are the components of language studied and connected to mathematics. One theory suggests phonological awareness directly facilitates the differentiation and manipulation of individual words in a number sequence when young children learn number words (e.g., one, two, three) and then indirectly influences mathematics in the later years via its relation to visual-spatial working memory and early mathematics skills (e.g., 5 is composed of 2 and 3 more; difference between 5 and 3 is 2; Krajewski et al., 2009). Phonological awareness predicts sequential counting in the informal numeracy phase of learning mathematics (prior to school entrance) and growth in calculation skills (Soto-Calvo et al., 2015). It has been suggested that phonological awareness influences the rate at which children acquire their first few number words much like it does for vocabulary acquisition (Soto-Calvo et al., 2015). Phonological sensitivity may support formal calculations via strategies, like growth in counting speed, which then influences the use of effective counting strategies (Soto-Calvo et al., 2015). Research also shows associations between phonemic awareness and numerical abilities such as magnitude processing on a number transcoding task (e.g., converting *six* to *6*) in children in grades 2 to 4 (Lopes-Siva et al., 2014). Phonemic awareness was found to mediate the influence of verbal working memory that is required for number transcoding (Lopes-Siva et al., 2014). This finding suggests an important pathway between the verbal input of numbers (e.g., eleven, eighty-one) and the Arabic output (e.g.,11, 81; Lopes-Siva et al., 2014).

Phonological processing is also suggested to be influential in the development of mathematical computation skills due to the speech sound processes utilized when a child solves a computation problem (Hecht et al., 2001). For example, when solving a problem like “6 x 4”, the terms (6, 4) and operator (multiplication) are converted into a speech-based code and then the phonological information is processed through either retrieving an answer code found in long-term memory or using a counting-based strategy in which the phonological name codes of numbers are used (Hecht et al., 2001). Hecht et al. (2001) investigated the relation between phonological processing and individual differences in mathematical computation skills and possible associations between three types of phonological processing abilities (phonological memory, rate of access to phonological name codes from long-term memory, and phonological awareness) and individual differences in reading and mathematical computation skills in children 92.5 to 134.8 months in age. Hecht et al. found evidence to support the assertion that there is a relation between individual differences in all three types of phonological processing abilities (which have been shown to influence growth in reading) and math computational skills. Some of these relations change over time and a bidirectional relation between general computation skills and simple arithmetic problem solving speed exists (Hecht et al., 2001).

Recently Purpura and colleagues (Purpura et al., 2011; Purpura & Napoli, 2015) examined both phonology (phonological processing) and semantics (vocabulary focus) with early numeracy. Results from studies applying this framework showed that each language component was related to, and predictive of, numeracy knowledge. The contribution of phonology and semantics to mathematical development in young children shows both are related to number naming, are a strong predictor of a child’s number line knowledge (1 to 1000), and account uniquely for the variability in different mathematical outcomes (e.g., numeration, calculation, geometry, measurement, magnitude comparison) depending on the task (LeFevre et al., 2010). This is a significant finding substantiating the hypothesis that language acts as pathway for learning symbolic mathematics (LeFevre et al., 2010). Semantics was uniquely predictive of later numeracy performance when nonverbal abilities and initial numeracy performance were taken into account (Purpura et al., 2011; Purpura & Napoli, 2015). They also found that oral language and numeral knowledge was fully mediated by informal numeracy knowledge. This supports the important role of oral language to early numeracy and for later performance in numeracy as well. It makes sense that semantic language plays an integral role in the learning and application of mathematics because there are a plethora of terms, vocabulary, functions, and content specific to the understanding and communication of math concepts and operations. This is often referred to as “math language” in the literature and involves both quantitative and spatial words and meaning crucial for mathematical development (Purpura et al., 2016; Purpura & Lonigan, 2015; Purpura & Reid, 2016; Toll & Van Luit, 2014).

Research on semantics and the development of mathematical knowledge and skills is extensive. Semantic language used in early mathematics can be quantitative (e.g., less than, more than, fewer, many), and spatial (e.g., beside, under, over, above). Semantic language has been identified in the literature as being a strong predictor and important to the development of math knowledge (i.e., early calculation skills and early numeracy skills; Purpura et al., 2016; Purpura & Lonigan, 2015; Purpura & Reid, 2015; Toll & Van Luit, 2014). Quantitative and spatial vocabulary is believed to facilitate a child’s ability to make and describe comparisons between numbers or groups, and promote the ability to understand and talk about the relations between objects (Purpura et al., 2016, 2017). For example, the term *less* can indicate a decrease in quantity (“I want less”) or it can be used to make comparisons (“8 is less than 10”), which serves to improve a child’s understanding of quantity (Peng & Lin, 2019; Purpura et al., 2016). Spatial language such as *before* and *after* assists with connecting sequences (“8 comes before 10”; Peng & Lin, 2019; Purpura et al., 2016). Access to a variety of quantitative and spatial language terms is believed to support the comprehension of math content that enables conceptualization and application (Purpura et al., 2016). One study showed that an 8-week intervention focused on quantitative and spatial mathematical language resulted in positive effects on general mathematical skills (Purpura et al., 2016).

The integral role of quantitative and spatial vocabulary for math performance has also been found, outside the alphabetic language system, in Chinese speaking children (Peng & Lin, 2019). Peng and Lin (2019) investigated whether the effects of quantitative and spatial vocabulary varied with the types of mathematical strand related vocabulary (e.g., longer vs. shorter; adjacent vs. above, add vs. subtract) or with different skills (e.g., calculation, word problems) as well as whether vocabulary mediated the relation between cognitive skills (i.e., working memory, non-verbal reasoning, processing speed) and mathematical performance. They found that mathematical vocabulary made unique contributions to performance on word problems, especially with respect to measurement and geometry, and numerical operations. Research shows that language predicts math performance irrespective of one's first language. McClung and Arya (2018) compared the Chinese language system to English numerical language on mathematics learning in children in grade 4 who had varying levels of reading and mathematics ability. They found that language significantly predicted mathematical performance even after math ability was accounted for, highlighting that not only did language have a positive influence, but that language was particularly important for children in the study who experienced mathematical difficulties compared to those that did not.

Similarly, a recent study investigated the role of early language abilities in the development of math skills in a large sample of young Chinese children (Zhang et al., 2017). These children were assessed for both informal math (skills learned prior to entering school) and formal math skills (learned through instruction), language abilities, and nonverbal intelligence.

Zhang et al. (2017) found that language abilities predicted formal math skill development, both directly and indirectly – findings that support the existing models of mathematic developmental pathways and precursors. Further, their research established that better vocabulary during the informal math phase influenced and supported better math performance - findings which are echoed in the English language as well (Peng & Lin, 2019; Purpura et al., 2016; Zhang et al., 2017). Collectively, the outcomes of this body of research hints to universal associations between semantics and mathematics that transcend the language (e.g., Chinese vs. English), lending credibility to the strength of the association between these two domains. This may present an avenue to explore how semantic language learning may be a valuable element to emphasize for children experiencing difficulties in mathematics.

There appears to be only one study examining syntax and mathematics. Carrerias et al. (2010) used fMRI to examine grammatical processing and associated brain activity in Spanish-speaking participants. They found that grammatical processing of phrases with gender and number agreement, and gender violations activated quantity processing areas of the brain (i.e., right intraparietal sulcus; left inferior frontal area). The findings of this research support the premise that language processing is complex and not restricted to the typical cortical areas associated with language (i.e., Broca or Wernicke). It also provides evidence that the brain regions associated with the processing of quantity or non-symbolic information (i.e., right intraparietal vs left intraparietal function) is associated with syntactic linguistic tasks (Carrerias et al., 2010). A common thread of each of the studies reviewed above is the underlying link and importance of oral language to all phases of symbolic mathematics. The opportunity to potentiate math learning via oral language can be started as early as infancy – long before the child develops other skills such as reading and writing. The value of investigating the association between oral language and the ANS cannot be discounted given how integral language is to the symbolic mathematical system.

Language and The Approximate Number System

In the current literature, the ANS is proposed to be independent of language (Dehaene et al., 1998; Nys et al., 2013). The premise for this model stems from phylogenetic (evolutionary) and ontogenetic (developmental) perspectives which suggest that ANS processing is innate, evident in infants, and predates verbal language and any formal teaching (Odic & Starr, 2018; Starr et al., 2013; Xu & Arriaga, 2007). This may explain why, to our knowledge, only five studies have investigated the relation between the ANS and language and only one of the five has examined this relation longitudinally. The five studies focused on literacy and symbolic and non-symbolic numerical processing (Zebian & Ansari, 2011), language impairment (Nys et al., 2013), restricted number vocabularies in indigenous groups (Butterworth et al., 2008; Pica et al., 2004), and vocabulary and mathematical language (amongst other variables) as predictors of the ANS (Purpura & Sims, 2018). These studies appear to support the proposal that the ANS is not associated with language, but this research is not without flaws.

Zebian and Ansari (2011) compared two groups with different literacy abilities (literate, $n = 11$ and minimally literate, $n = 11$) in order to examine whether literacy skills (reading comprehension, writing skills, and grammatical knowledge) were associated with symbolic and non-symbolic representations of magnitude in male adults. They found that the two groups did not differ with respect to accuracy on the tasks but that the numerical distance effect (as measured by steepness of slope) differed between the literate and minimally literal groups for the symbolic but not the non-symbolic magnitude comparison task. Zebian and Ansari contend that their findings indicate that literacy and education impact the symbolic and non-symbolic processing system in different ways. There are several problems with this study, however. The difference between the two groups is represented by a subtle difference in the magnitude of the distance effect and not a categorical difference between the two groups. Additional problems include a small sample size, male participants only, and lack of consideration of IQ. Furthermore, although word reading can be considered a language-based skill, it does not represent oral language competence, which was not measured. As a result, this study does not provide any convincing support for the argument that the non-symbolic system operates independently of language.

Nys et al. (2013) examined language and exact and approximate number skills in 28 children aged 7 – 14 years with specific language impairment (i.e., poor oral language skills with average intellectual ability and no neurological or hearing impairments), separated into a younger (7-10 years; n = 15) and older (11- 14 years; n =13) group. Control groups of children matched for chronological age and vocabulary age were also included. Children were given symbolic and non-symbolic tasks (counting, written and mental exact arithmetic, number comparison, approximate arithmetic, approximate comparison). In both age groups, children with specific language impairment showed worse performance than control children on the exact symbolic arithmetic task, but not on the approximate number tasks (after accounting for differences in cognitive abilities). Nys et al. concluded that approximate number skills were preserved in children with specific language impairment and that language was not important for approximate number skills. However, there are several problems with this conclusion. Language is associated with the cognitive abilities that were measured in this study (e.g., executive function) and it is possible that the authors removed some of the variance associated with language in their analyses. Although the authors examined whether language was associated with approximate number skills in the groups with specific language impairment, they did not examine relations between language and approximate numbers skills in the control group. In addition, although vocabulary and morphosyntax was assessed, a more fine-tuned examination of language skills is required (e.g., relational vs. nonrelational vocabulary; syntax) to determine whether language is truly not required for approximate number skill. Finally, based on an age effect of better performance for older children and lack of difference in performance between the older control and impaired groups, Nys et al. proposed that experience and development may enhance approximate number skills during childhood. Since this is a cross-sectional and not longitudinal study, this interpretation is flawed. Using cross-sectional research to make conclusions regarding development in atypical groups is problematic. There are other reasons older children with specific language impairment could perform at age expected levels on tasks assessing approximate arithmetic (e.g., language interventions that have a positive collateral impact) that need to be examined with longitudinal methods.

Butterworth et al. (2008) and Pica et al. (2004) examined language and exact and approximate concepts of number processing in adult and child indigenous groups (Amazonian and Australian, respectively) whose language includes limited words for exact quantities (e.g., “one or two” opposed to one, two, or three). The researchers questioned whether the concept of exact numbers can exist without the words to represent them, and hypothesized that without exact number words, only approximation could take place (e.g., few or many). Butterworth et al. compared exact and approximate concepts of number in English (n = 13) and Indigenous (Warlpiri n = 20; Anindilyakwa n = 12) speaking children aged 4 – 7 years. Children were given four enumeration tasks to examine numerosity understanding (memory for number counters up to 9, cross-modal matching of discrete sounds and counters up to 7, nonverbal exact addition up to 8/sharing task up to 10). Age rather than language accounted for differences in performance (i.e., there were no performance differences between the different language groups). Butterworth et al. concluded that the numerical competence is not dependent on numerical vocabulary but rather, conceptual development is needed before counting words are acquired. Although we do not disagree with the author's conclusion that conceptual development is important for acquiring number concepts, the findings do not necessarily support the premise that the ANS is a separate system. Although counting vocabulary is important for enumeration, it is possible that the children used other language skills for numerosity. More extensive language skill examination is required to assess any effect on ANS performance.

Pica et al. (2004) examined numerical cognition in speakers of an Indigenous language (Munduruku) with number words for one to five only. Pica et al. proposed that exact mathematic calculations beyond five would be affected. They controlled for educational instruction and exposure to other languages/cultures and contrasted performance on approximate and exact arithmetic tasks between four groups of Munduruku speakers with a control group of native French speakers (n = 10). Participants (whose average age was 50) were given four tasks (magnitude comparison, approximate addition and comparison, nonverbal and verbal exact subtraction tasks). Pica et al. reported clear differences on the exact calculation task, but uneven performance on the approximate calculation tasks and surmised that there must be a distinction between a nonverbal system of number approximation. On average, the Munduruku speakers performed slightly worse than the control group on the magnitude comparison but comparably on the approximate addition and comparison tasks. They concluded that a lexicon of number words was needed for exact number and calculation and that their data supports the premise that the ANS is an innate ability, independent of language. However, this view is highly problematic because they did not actually assess language skills (other than the observation of the use of number words when presented with 1 to 15 dots) to rule it out as a contributor to number approximation and language was not absent in the indigenous speaking participants. What Munduruku speakers appear to lack is a formal counting routine and number vocabulary, which are skills arguably learned through exposure, instruction, modeling, and practice. As with the other research highlighted in this section, the study design limits generalizability and the reliability is questionable.

Purpura and Sims (2018) examined 113 preschool aged children (3 to 5 years) at two time points (fall and spring) within one academic year in order to investigate the stability and predictors of the ANS as a step to better understanding the ANS relation to early mathematics performance. Purpura and Sims were interested in examining whether general cognitive factors (working memory, executive functions) and specific math related skills (ANS, counting skills, calculation fluency, and mathematical language) known to contribute to mathematical skills may be related to ANS performance. They hypothesized that general cognitive factors and specific math skills would account for significant variance in predicting ANS performance by the end of preschool (i.e., in a fall vs spring comparison). A large battery of measures were given, but the language measures included an expressive vocabulary test and a mathematical language test (developed by the author) that consisted of 16 comparative and spatial language words. Results showed that ANS performance in the fall (beginning of preschool) was the strongest predictor of spring ANS performance (end of preschool), but that cardinality and response inhibition were also predictors. These findings contradict previous claims that the ANS is a foundational and causal building block for numerical skills and Purpura and Sims suggest that ANS precision may be influenced and shaped by other cognitive functions, as well as school experience. A weakness of this research, however, is that Purpura and Sims did not control for reliance on conceptual cues (i.e., size of stimuli, which can impact judgement of quantity or numerosity) and call into question whether ANS processing was actually measured. In addition, the participants ranged in aged from 3 to 5 years resulting in a large difference in math skills (e.g., counting to 10 vs. knowing simple addition, understanding math language such as “big” and “small”, and sequencing such as first and second). This makes it difficult to isolate the association between general cognitive abilities and specific math abilities to the ANS at a specific age and point of development. Additionally, the assessment of language is narrow and focused on vocabulary only.

What is absent in any discussion of the literature reviewed above is that when contrasting groups on the basis of number lexicon knowledge, we must acknowledge that from birth, all humans are exposed to oral and pragmatic language. To assume language is not influential, directly or indirectly, on the non-symbolic system because a child or adult demonstrates an apparent lack of a honed lexicon for numbers is flawed. In summary, there is little empirical data that has specifically focused on the potential contribution of language on the ANS and the differing methodologies in the existing research does not allow direct comparisons of the findings in these studies. To our knowledge, there are no studies that have specifically looked at different aspects of language and the ANS in children, using a longitudinal approach. This is needed to determine if associations exist between language and the ANS over time and to characterize what they might be.

The Current Study

The literature reviewed above shows language is a strong predictor of early mathematical success (LeFevre et al., 2010; Purpura et al., 2011) and associated with the symbolic system

(Hyde et al., 2014; Libertus et al., 2011). Research on the ANS and the brain activity of developing young children show shared functions between symbolic math processing and language that also support ANS processing, directly or indirectly. Taken together, this suggests that early oral language skills may support later ANS processing, which holds potential for early education/pedagogy in areas of both language and math as a method for improving math understanding and performance. For example, oral language assessments in the preschool or kindergarten years could identify children who might benefit from early intervention and targeted support in math language learning. The current study provides a preliminary exploration of whether different oral language components (phonology, morphology, syntax, semantics, relational concepts, and verbal memory) relate to ANS processing concurrently and over time. Based on the review above, it is reasonable to expect both concurrent and predictive relations between oral language and ANS processing in 4-year-old children and one year later when they are 5 years of age.

Method

Participants

The participants of this current study are comprised of 26 children - a subsample of children enrolled in a longitudinal study investigating the relations among language, numeracy, and executive function. Recruitment took place at Peterborough and Toronto area daycares. Parents of all children turning 3-years-old were provided with information about the study and an opportunity to provide written consent for their children to take part. Parental consent was obtained from all participants. For the current study, data from 4.0 and 5.0 years was used. Only children who had estimated IQ within the average range (80 to 120) and quantity discrimination data for both time points were included.

Procedure

At 4.0 years children completed measures of intelligence, oral language, verbal memory, and quantity discrimination. Children also completed other measures not relevant for the current study. At 5.0 years children completed measures of oral language, verbal memory, and quantity discrimination as well as other measures not relevant for this study. Test sessions took place at the child's respective daycare, school, or family dwelling setting. Children engaged in three to five individualized testing sessions depending on child attention and engagement, with breaks as needed. Child verbal assent was obtained prior to each test session.

Measures

Intelligence

Four subtests: *Information, Similarities, Block Design, Matrix Reasoning* from the *Wechsler Preschool and Primary Scale of Intelligence – Fourth Edition* (Wechsler, 2012) were used to estimate IQ and control for cognitive abilities. The *Information* subtest measures the capacity to acquire, retain, and retrieve general factual knowledge. The *Similarities* subtest assesses the child's ability to describe how two objects or concepts are similar. The *Block Design* subtest measures the capacity to analyze and synthesize abstract visual patterns and the *Matrix Reasoning* subtest requires the child to select one of four possible stimuli that fit into a sequence or pattern. The standardized scaled scores from the four subtests were used to calculate an overall standardized scaled score used to estimate IQ.

Oral Language

The phonological, morphological, syntactic, and semantic aspects of oral language were measured. Selected items from the *Early Reading Skills* (*ERS*) subtest of the *Wechsler Individual Achievement Test – Third Edition* (Wechsler, 2010) was used to assess phonology when children were 4 years and 5 years. Thirteen items were chosen because they measured oral phonology (e.g., rhyming; listening for similar beginning and ending word sounds) that require pointing responses. Score is total items correct out of 13, which was converted to a proportion.

The *Clinical Evaluation of Language Fundamentals – Preschool: Second Edition* (Wiig, et al., 2004), a standardized measure of oral language, was used to assess morphology, syntax, and semantics when children were 4 years and 5 years. Morphology was measured with the *Word Structure* subtest, which assesses comprehension of word structure rules (e.g., inflectional, derivational, or comparative and superlative suffixes; referential pronouns). Syntax was measured with the *Sentence Structure* subtest, which evaluates the ability to comprehend spoken sentences of increasing length and complexity. Semantics was measured with the *Word Classes, Expressive Vocabulary, and Basic Concepts* subtests. *Word Classes* requires the ability to group words that are in a similar category and *Expressive Vocabulary* requires the ability to provide labels of people, objects, and actions. The *Basic Concepts* subtest evaluates knowledge of concepts of dimension/size, direction/location/position, number/quantity, and equality. Each subtest provides a standard scaled score.

Each language standard scaled score was converted to a z-score. A mean composite score was created for semantic language using the z-scores for the three subtests that measured semantics listed above.

The *Test of Relational Concepts* (Edmonston & Litchfield Thane, 1999), a standardized measure, was used to assess the area of semantics specific to relational concepts that include: temporal (first/last), quantitative (more/less), dimensional (big/little), and spatial (front/back) terms. This test provides a standard scaled score. The standard scaled score was converted to a z-score.

Verbal Memory

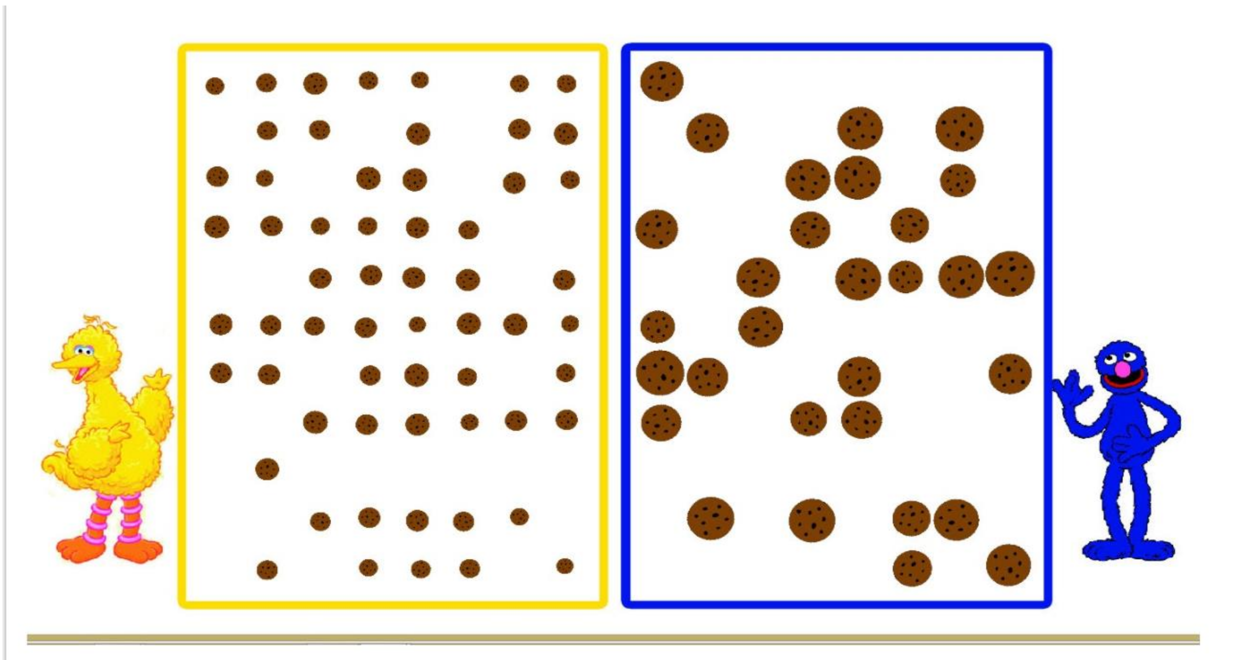
Verbal memory was assessed with the *Recalling Sentences* and *Concepts & Following Directions* subtests from the *Clinical Evaluation of Language Fundamentals– Preschool: Second Edition*. The *Recalling Sentences* subtest requires the child to repeat sentences of increasing length and the *Concepts & Following Directions* subtest measures the ability to execute oral directions of increasing length and complexity. Standard scaled scores are provided for these two subtests, which were converted into z-scores. A mean composite verbal memory score was created using the z-scores of the two subtests.

Approximate Number System (ANS)

The ANS was measured with the *quantity discrimination task*, using EPrime software, and based on descriptions in Halberda and Feigenson (2008) and Libertus et al. (2011, 2013). The quantity discrimination task is considered the most reliable measure of ANS acuity with less demand on working memory (Dietrich et al., 2015). The quantity discrimination task used in the current study was delivered on a notebook computer with an external 14-inch touchscreen. Participants sat approximately 40 cm from the screen. Our task was unique in design compared to other studies in that it included a greater breath of quantities divided into two versions: a small quantity version that includes 5 to 16 objects that resemble chocolate chip cookies and a large quantity version that includes 30 to 70 cookies. Each version included 36 items. In both versions, two arrays of cookies are presented simultaneously on the left and right side of the computer screen in different ratios (e.g., 1:2) for comparison. Arrays were designed with sets of five or greater to prevent subitizing. Each array appears within a line frame of yellow (left side) and blue (right side) to correspond to a static image of a small character (Big Bird in yellow or Grover in blue) placed on the lower outer margin of each frame (see Figure 1 for an example).

Children were asked, “Who has more cookies?” and to indicate Big Bird or Grover by touching the screen.

Figure 1
Trial from the Large Quantity Version of the Quantity Discrimination Task with Ratio of 1:2



Note: This trial from the large quantity version displays a ratio of 1:2 (60 cookies: 30 cookies). The correct answer to the question, “Who has more?” is Big Bird.

The difficulty of a quantity discrimination task depends on the ratio between the two array quantities – the closer the ratio the more difficult it is to discriminate between the sets. For example, the larger quantity in a ratio of 1:2 objects is easier than a ratio of 5:6 objects (Halberda & Feigenson, 2008). The following nine ratios, in order of least difficult to most difficult, were used for both versions: 1:2, 3:5, 2:3, 5:7, 3:4, 4:5, 5:6, 6:7, and 7:8. Four items for each of the nine ratios were displayed for a total of 36 trials. In the small quantity version, for example, a 1:2 ratio could include 5:10, 10:5, 6:12, 12:6, 7:14 or 14:7 items. In the large quantity version, a 1:2

ratio could include 30:60, 60:30, 35:70 or 70:35 items (see Appendix) for items included in each ratio for the small and large quantity versions). The four items that were presented for each ratio was randomly chosen by the program. For example, four items for the 1:2 ratio could be 14:7, 5:10, 5:10, and 5:10, or 10:5, 12:6, 7:14, and 6:12. Randomization could result in repeated or reversed items (e.g., 60:30; 30:60).

In order to control reliance on perceptual cues for quantity discrimination judgement, the area taken up by the cookies was controlled for in each version (small and large quantity) with half the trials being area correlated and the other half being area anticorrelated. In the area correlated trials, the total area taken up by the cookies was equal on both sides regardless of the ratio of the array. For example, a 1:2 array includes twice the number of cookies on the right side (e.g., 8) compared to the left side (e.g., 4), yet the total area covered by the cookies was equal on both sides. In the area anticorrelated trials), the area covered by the cookies was reversed so that the side with fewer cookies would cover more area, in an amount equal to the inverse of the ratio being represented. For example, a 1:2 array could display 4 cookies on the left side and 8 cookies on the right side, but the 4 cookies occupied twice the total area of the 8 cookies. In addition, to prevent children from relying on the cookie size as a cue for quantity, the size of each cookie in each array across all trials varied +/- 35% of the average cookie size displayed in a particular array. This resulted in each array consisting of cookies of different sizes (larger or smaller) by a maximum of 35% relative to the average cookie size displayed in the particular array.

Each version of the task began with eight practice trials that included arrays with a 1:2 ratio. Children were told that they had a short time to decide who had more cookies before the cookies disappear. Children moved on to the test trials only if they understood the task instructions. Practice trials were repeated as required. Each practice or test trial began with a centered fixation point lasting 1000 ms followed by an array for 1200 ms to prevent counting. Empty yellow (Big Bird) and blue (Grover) frames were then displayed until a response was recorded, at which point the next item appeared. Children could respond when the arrays were displayed or after they disappeared. Correct and incorrect responses were indicated by different audible tones. Both scores for correct and incorrect responses, as well as response time, were recorded by the computer. Proportion of correct responses were used for analyses in keeping with the procedure design of Halberda and Feigenson (2008).

As mentioned above, ANS acuity is represented by an individual's ability to consistently identify the smallest difference between two quantities (Bonney & Lourenco, 2013; Chen & Li, 2014; Halberda & Feigenson, 2008; Odic & Starr, 2018; Wang et al., 2018). A widely used numerical measure of this acuity is the Weber fraction (*w*; Chen & Li, 2014; Grantham & Yost, 1982; Halberda & Feigenson , 2008; Park & Starns, 2015), which is calculated by identifying the difference between the two closest ratios (e.g., 4:5 vs 5:6) among the range being compared (i.e., 1:2 to 7:8) where a noticeable difference in correctness better than guessing (50%) is achieved by the participant (e.g., 75% on 4:5 ratios vs 50% on 5:6 ratios). However, as noted in the literature, overall accuracy (percentage correct; Geary & VanMarle, 2016; Honore & Noel, 2016; Wang et al., 2016) is a more viable and reliable measure of ANS acuity in young children. Proportional scores of total accuracy were calculated after being presented with four sets of the nine ratios (36 trials) for each version of the task (i.e., small quantity, large quantity)

Results

Data Screening

Missing value analysis was conducted across all measures. Two cases of missing data in phonology were found for one child at age 4 and another at age 5. This was considered problematic due to the small sample size and the consequential influence missing data had on the mean and standard deviation. Imputation using the expectation maximization technique was used to replace the missing values (Dong & Peng, 2013)

The means, standard deviations, range, and skewness values for oral language, verbal memory, and quantity discrimination data were considered at both time points and all were as expected and within the acceptable norms. However, due to the small sample size, an analysis of normality using a Shapiro-Wilk test for each variable was performed. The Shapiro-Wilk test showed that all language distributions were not significantly departed from normal. Univariate outliers were present for the following: At age 4, there was one outlier for morphology and two outliers for syntax, as well as six outliers for the quantity discrimination task small quantity version and one for the large quantity version. At age 5, there were three outliers for morphology and one outlier for the quantity discrimination task large quantity version. All cases were retained. Paired sample *t*-tests were conducted to examine group differences and non-parametric analyses were conducted for correlations due to the outliers.

Descriptive Statistics

There were 26 participants, 13 boys and 13 girls. Means, standard deviations, and results of paired sample *t*-tests for age, oral language, verbal memory, and quantity discrimination between the two time points (4 and 5 years of age) are reported in Table 1.

Paired sample *t*-tests indicated significant improvement in performance between 4 and 5 years in phonology and relational concepts. There were no significant differences between age 4 and 5 years on morphology, syntax, semantics, verbal memory, or quantity discrimination on the small quantity version. However, children showed significantly improved performance between 4 and 5 years on quantity discrimination for the large quantity version.

Table 1.
Descriptive Characteristics and Differences for Age, Estimated IQ, Language, Verbal Memory, and Quantity Discrimination at Age 4 and Age 5 (n = 26)

	Age 4		Age 5		*t*	*p*	*d*
	M	*SD*	*M*	*SD*			
Age (months)	48.50	.65	60.19	.75	-56.81	.001	1.05
Estimated IQ	99.58	12.24					
Language							
Phonology	.25	.15	.49	.20	-6.72	.001	.19
Morphology	-.01	.82	-.17	.69	.91	.37	.86
Syntax	.10	.79	.10	.85	.00	1.00	.88
Semantics	.20	.58	-.08	.67	1.70	.10	.83
Relational Concepts	.03	.71	.51	.65	-3.38	.002	.71
Verbal Memory	.04	.85	-.12	.73	1.03	.31	.79
Quantity Discrimination							
Small Quantity Version	.55	.12	.59	.12	-1.21	.24	.16
Large Quantity Version	.54	.12	.61	.14	-2.24	.03	.15

Note: Estimated IQ = Standardized scaled score; Phonology = mean proportion score; Morphology, Syntax, Semantics, Relational Concepts, and Verbal Memory scores = z-scores; Quantity Discrimination scores = mean proportion score.

Correlational Analysis

The Spearman test was used for correlational analyses due to the extreme outliers in some of the variables. See Table 2 for correlations.

Language Skills

At age 4, morphology and relational concepts, syntax and relational concepts, and semantics and relational concepts were significantly positively associated. The association between morphology and relational concepts was not present a year later at age 5. However, at age 5, phonology and syntax, phonology and relational concepts, syntax and semantics, syntax and relational concepts, as well as semantics and relational concepts, were significantly positively associated.

Table 2.
Non-Parametric Correlations Between Language, Verbal Memory, and Quantity Discrimination at Age 4 (top diagonal) and Age 5 (lower diagonal; n = 26)

Variable	1	2	3	4	5	6	7	8
1. Phonology	-	.14	.36	.31	.31	.41*	.13	.09
2. Morphology	.38	-	.22	.31	.43*	.35	.45*	.34
3. Syntax	.40*	.36	-	.16	.43*	.54**	.09	.29
4. Semantics	.10	.14	.48*	-	.55**	.29	.55**	.14
5. Relational Concepts	.44*	.20	.52**	.52**	-	.57**	.40*	.44*
6. Verbal Memory	.22	.69**	.45*	.28	.32	-	.05	.37
7. Small Quantity Set	.23	.31	.11	.24	.28	.24	-	.37
8. Large Quantity Set	.22	.30	.44*	.59**	.72**	.34	.33	-

**$p < .01$(2-tailed)
*$p < .05$ (2-tailed)

Language and Verbal Memory

At age 4, verbal memory showed significant positive associations with phonology, syntax, and relational concepts. A year later, a significant positive association remained between verbal memory and syntax. In addition, verbal memory was significantly positively associated with morphology.

Quantity Discrimination

Interestingly and unexpectedly, the small quantity and large quantity versions were not significantly associated at either time point. Recall that the same ratios were presented in each version; the only difference between the two versions was the number of dots in each one (small = 5 to 16; large = 30 to 70). This result suggests the two tasks are conceptually different in the current sample. See appendix B for quantity discrimination average percent correct per ratio by task version, and age.

Language, Verbal Memory, and Quantity Discrimination

At age 4, morphology, semantics, and relational concepts were significantly positively associated with the small quantity version and relational concepts was significantly associated with the large quantity version. At age 5, language was not correlated with the small quantity version; however, semantics, relational concepts, and syntax predicted accuracy on the large quantity version.

Relations between Oral Language at Age 4 and Quantity Discrimination at Age 5

It was expected that oral language at 4 years would be correlated with ANS processing (as measured by the quantity discrimination task) at 5 years of age. See Table 3 for correlations. At age 4, oral language and verbal memory were not significantly associated with the small quantity version of the quantity discrimination task at age 5. However, morphology, syntax, and relational concepts as well as verbal memory at 4 years of age were significantly positively associated with the large quantity version of the quantity discrimination task at 5 years.

Table 3.
Non-Parametric Correlations between Language at Age 4 and Quantity Discrimination at Age 5 (n = 26)

	Quantity Discrimination at 5 years	
Language at 4 years	Small Quantity Version	Large Quantity Version
1.Phonology	.06	.27
2. Morphology	.10	.43*
3. Syntax	.15	.49*
4. Semantics	.28	.32
5. Relational Concepts	.35	.53**
6. Verbal Memory	.39	.66**

**$p < .01$ (2-tailed)
*$p < .05$ (2-tailed)

Discussion

The current study is a preliminary investigation of the concurrent and longitudinal relations between different aspects of oral language and the ANS (quantity discrimination) in 4-year-old children and one year later when they turn 5. As hypothesized, we find that aspects of oral language are associated concurrently with quantity discrimination at age 4 and age 5. In addition, aspects of oral language at age 4 predict quantity discrimination (large quantity version) one year later.

Concurrent Relations between Language and ANS Performance

At age 4, morphology and semantics are correlated with ANS performance (small quantity version). Also at age 4, relational concepts predict ANS performance (small and large quantity version). At age 5, syntax, semantics, and relational concepts are positively associated with ANS performance (large quantity version). These findings contradict the idea that the ANS is a language independent system (Feignenson et al., 2004; Lindskog et al., 2013) and suggest a dynamic relation between components of oral language and ANS performance.

Notable differences in the language components associated with quantity discrimination is the use of morphology at age 4, which is absent a year later, and syntax which comes into play at age 5. Conceptually this might be explained by the difference in the conceptual complexity between morphology (word structure level) and syntax (sentence level). Recall that morphology involves how words may be inflected to express grammatical categories such as number, tense, and aspect. For example, we come to understand that the suffix “er” on “big” changes the meaning to indicate a larger quantity. Similarly, the suffix “er” on “small” changes the meaning to a less than small quantity, giving a more refined sense of quantity discrimination. Conceptually, word structure may scaffold the understanding of quantities in relation to one another. This helps to explain why relational concepts, a specific aspect of morphological knowledge that focuses on the relation between two items, is also correlated to quantity discrimination at age 4.

Recall that syntax, refers to language structure rules at a higher level - phrases, clauses, or sentences. Active discrimination of a larger quantity of items may require different processes because of its greater magnitude, which may not allow perceptual strategies (e.g., larger area) or basic strategies such as counting. Thus, the child may rely on other problem-solving skills to make a judgement. Syntax may be useful conceptually because the challenge of comparing, interpreting, and estimating two large quantities side by side is spatially and conceptually similar to comparing and interpreting adjacent words at a sentence level. Interpreting meaning through syntax may be advantageous to discerning larger quantities in this way.

Consistent at both age 4 and 5 is the association of quantity discrimination with semantics and relational concepts. Both aspects of language increase a child's understanding of concrete and abstract concepts, with relational concepts being specific to quantitative and spatial language. Semantics provides an appreciation of different shades of meaning in words and phrases. For example, the ability to distinguish a transit bus from a school bus means being able to differentiate between these two large multi-passenger vehicles. This practice of discerning differences between concepts is a similar cognitive exercise as the quantity discrimination task. Understanding relational concepts provides focused knowledge regarding the ordering of quantities and prompts the utilization of any prior mathematical knowledge or practice (i.e., more/less; estimation) and apply it (Purpura et al., 2016). It is plausible that the ability to accurately detect differences in quantity is aided by general semantic understanding as well as a more specific understanding of relational concepts such as more and less than.

Of interest is the changing association between language and ANS performance with quantity of stimuli. Recall that the small and the large quantity versions have a different number of dots within each ratio. The small quantity version ranged between 5 to 16 dots and the large quantity version ranged between 30 to 70 dots. These findings suggest that different components of oral language provide conceptual knowledge (or tools) that children can draw upon to assist in discerning quantity, depending on the magnitude of the task (small quantity vs. large quantity). Moreover, neuropsychological research examining language and the ANS has shown that syntactic and semantic language task prime and activate the same brain areas utilized for quantity discrimination (right intraparietal sulcus; Carreiras et al., 2009). This suggests that quantity discrimination relies on multiple language resources.

Relations between Language at Age 4 and ANS Performance at Age 5

At age 4, morphology, syntax, relational concepts, and verbal memory predict ANS performance on only the large quantity version at age 5. This is in contrast to the findings regarding concurrent relations between language and ANS performance in the current study. In particular, verbal memory predicts ANS performance one year later. Verbal memory involves the practice of recalling words, verbal items, and/or language-based memory that in some circumstances should help when holding in mind two items for comparison. Experience using language like “more than” or “greater than” may verbally scaffold comparison of two different items of large quantity. In a quantity discrimination task with a large number of items, verbal memory may help to scaffold comparison of large quantities versus small quantities (which may not require verbal memory). These results provide additional evidence for the role of oral language in the development of quantity discrimination and also suggest that the components of language relied upon for the operation of the ANS is different for small quantities compared to large quantities performance. To our knowledge, no other study has included the wide range of quantities for comparison that we have across the two versions.

As previously discussed, to our knowledge, only five studies have focused on some aspect of language and ANS performance, and only three of these studies were with children (3 to 5 years; 4 to 7 years; 7 to 14 years). The current study is the first to compare oral language with ANS performance in very young children longitudinally over more than one year of development. Our findings indicate that the ANS may not be language independent and that oral language may play a role in ANS performance.

Practical Implications

Although our findings must be replicated, they have important recommendations for education and adapting curricula in the early years of childhood. There is considerable research that has established that improving language skills can also improve mathematics achievement and that language is a strong predictor of early mathematical success. Now, we have new insight into the potential relation between language and the ANS – the sister construct of symbolic math and the foundation for later math cognition. This potential relation suggests an earlier time frame within child development for building up the ANS through oral language. Early childhood educators could develop purposeful and targeted oral language instruction that has the collateral effect of improving the ANS. This instruction could have a dynamic focus on different components of language and verbal memory and be modified for age as suggested by the current study. For example, since we find morphology, semantics, verbal memory, and relational concepts are important at age 4, curricula, resources, and instruction could focus on integrating development in these areas. At age 5, the language focus broadens to includes syntax and curricula design could reflect this accordingly.

Our findings also suggest that oral language should be screened and monitored in young children in a comprehensive mathematics development program that uses oral language as a starting place for early intervention. As previously discussed, the ANS is thought to be the foundation for more sophisticated symbolic mathematics and its acuity in early childhood is correlated with later performance in mathematics (Bonny & Lourenco, 2013; Chen & Li, 2014; Fazi et al., 2014; Halberda & Feignenson, 2008; Libertus et al., 2011; Mazzocco et al., 2012; Wang et al., 2016). Moreover, since improving ANS acuity through educational training has positive effects on the ANS and symbolic mathematics performance (Honore & Noel, 2016; Odic & Starr, 2018; Park & Brannon, 2013), it follows that early years education (preschool, kindergarten, grade one) could effectively target oral language instruction and screening to support the ANS and overall mathematics development.

Study Limitations and Directions for Future Research

As with any longitudinal study, we experienced participant attrition one year later as children turned 5 years of age. Further the COVID pandemic interrupted data collection. As a result, the current study has a small sample size that impacts generalizability and power. Another possible consequence of our small sample was the occurrence of extreme outliers that limited our ability to carry out parametric measures. Consequently, our study may be considered a positive first step in understanding the contribution of oral language to the ANS (believed to be language independent) that requires further research with a larger sample. Additional work examining different components of oral language would provide important information regarding the contribution of language as cognitive demands for quantity discrimination. Our study focused on two time points of development, one year apart; however, further inquiry regarding oral language and the ANS is required over a greater span of development to explore the relationship over time. The measurement of ANS performance via quantity discrimination in young children proved a further challenge. The quantity discrimination task (Halberda & Feigenson, 2008; Libertus et al., 2011, 2013) is considered the most reliable measure of ANS acuity with few demands on working memory (Dietrich et al., 2015). However, we were unable to identify a single consistent threshold where participants could discern the larger quantity in either the small or large quantity version of the task. This made it impossible to calculate a Weber fraction measurement of participant performance. Despite literature purporting Weber fraction calculation to be the most common measure of acuity, due to the more variable responses that the children in the current study generated, it was impossible to calculate. Instead, we used overall accuracy (percentage correct) as per Geary and vanMarle (2016); a score supported in the literature as the most viable and reliable for children. It is possible then that there is no finite minimum threshold of ratio detection for the ANS in children and that this threshold is variable. Moreover, the quantity discrimination task used in our study – designed with two different versions where children compare ratios using small quantities and large quantities, suggest different oral language resources may be drawn upon for approximation with different quantities. This is an area worth greater investigation in future research.

Conclusions

The current study is a preliminary investigation into concurrent and longitudinal relations between different aspects of oral language and the ANS in 4-year-old children and one year later when they were 5. Contrary to the current theory that suggests the ANS works independently of language, we find a dynamic set of predictive relations between oral language and the ANS. Correlational analysis show that these relations change over time and differ depending the magnitude of the quantity comparison (small versus large quantity task versions) and the specific oral language resources needed as conceptual tools to make this comparison. Collectively, our research findings provide preliminary evidence for the role of oral language in the development of quantity discrimination. Our findings also have positive implications for early childhood education and suggest opportunities for early years intervention and improvement of quantity approximation skills. Future research is warranted in order to fully explore and refine what is known about language and the ANS.

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Appendix A

Quantity Discrimination Task Ratio Descriptions

Small Quantity Discrimination Task Version for the Quantity Range 5 to 16

Ratio Possible Number of Dots on the Left Side and Right Side of the Screen

1:2	5:10	6:12	7:14	8:16
3:5	6:10	9:15		
2:3	6:9	8:12	10:15	
5:7	5:7	10:14		
3:4	6:8	9:12	12:16	
4:5	8:10	12:15		
5:6	5:6	10:12		
6:7	6:7	12:14		
7:8	7:8	14:16		

Large Quantity Discrimination Task Version for the Quantity Range 30 to 70

Ratio Possible Number of Dots on the Left Side or Right Side of the Screen

1:2	30:60	31:62	32:64	33:66	34:68	35:70				
3:5	30:50	33:55	36:60	39:65	42:70					
2:3	30:45	32:48	34:51	36:54	38:57	40:60	42:63	44:66	46:69	
5:7	30:42	35:49	40:56	45:63	50:70					
3.4	30:40	33:44	36:48	39:52	42:56	45:60	48:64	51:68		
4:5	32:40	36:45	40:50	44:55	48:60	52:65	56:70			
5:6	30:36	35:42	40:48	45:54	50:60	55:66				
6:7	30:35	36:42	48:56	54:63	60:70					
7:8	35:40	42:56	49:56	56:64						

Appendix B

Quantity Discrimination Average Proportion Percentage per Ratio by Task Version, and Age

Ratio	1:2	3:5	2:3	5:7	3:4	4:5	5:6	6:7	7:8
Age 4 Small Version	53.85	70.19	50.00	52.88	53.85	64.42	53.85	55.77	52.88
Age 5 Small Version	65.38	61.54	58.65	51.92	60.58	56.73	56.73	65.38	54.81
Age 4 Large Version	64.42	61.54	53.85	52.88	50.96	53.85	57.69	48.08	53.85
Age 5 Large Version	72.12	71.15	72.12	66.35	59.62	61.54	56.73	59.62	52.88